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A Novel $\mathcal{O}(n)$ Numerical Scheme for ECG Signal Denoising

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Abstract
High quality Electrocardiogram (ECG) data is very important because this signal is generally used for the analysis of heart diseases. Wearable sensors are widely adopted for physical activity monitoring and for the provision of healthcare services, but noise always degrades the quality of these signals. This paper describes a new algorithm for ECG signal denoising, applicable in the contest of the real-time health monitoring using mobile devices, where the signal processing efficiency is a strict requirement. The proposed algorithm is computationally cheap because it belongs to the class of Infinite Impulse Response (IIR) noise reduction algorithms. The main contribution of the proposed scheme is that removes the noise's frequencies without the implementation of the Fast Fourier Transform that would require the use of special optimized libraries. It is composed by only few code lines and hence offers the possibility of implementation on mobile computing devices in an easy way. Moreover, the scheme allows the local denoising and hence a real time visualization of the denoised signal. Experiments on real datasets have been carried out in order to test the algorithm from accuracy and computational point of view.

Keywords: Numerical Algorithms for Signals Denoising, ECG Signal, IIR Filters, Fast Fourier Transform

1 Introduction

The accurate analysis of noisy Electrocardiogram (ECG) data is a very interesting challenge. This is especially true in relation to the pervasive use of wearable healthcare monitoring systems [21], where physiological data acquired from real life can be used for remote healthcare scenarios, for the early analysis of diseases (e.g. [22]), or for highlighting of correlations between health and a correct lifestyle (e.g. [23]). The ECG biomedical signal (see Figure 1) is composed of weak non-stationary data which are affected by various types of noises: power line interference, baseline

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drift, electromyography interference and sensor contact noise. Generally, a good denoising scheme has the capability of removing noises, from the acquired signal, by filtering the data and by ensuring a result as close as possible to the unknown original signal. In literature, there are numerous research papers devoted to this problem, including: adaptive filtering [24, 20], Wiener filtering [5], Empirical Mode Decomposition [2], and wavelet denoising [19] (for other methods see [13]). However, these approaches are suitable for desktop use, but are not optimized for mobile use. This is a strong limitation, because, as documented in [15], healthcare systems are undergoing a mutation. In fact, the new technologies enable new channels for delivering health services, moving these to mobile health services. The Internet, tablets, smartphones, remote monitoring, and wireless applications and devices, including wearable devices such as a wrist watch or a small bandage that can monitor patients continuously [3], contribute to the building of new mobile systems in a competitive and rapidly changing environment. In this new scenario of health care, new approaches based on novel algorithms and smart software framework are widely explored for improving mobile health care (m-healt) services especially for recording and filtering biomedical signals [16, 6, 18]. In the contest of real time signal processing, two new aspects are demanded, the computational efficiency of the algorithm and the ability of local signal processing. In this paper, starting from a methodology based on Recursive Filtering, applied successfully by the authors in another research field [27, 8, 9, 10], we propose a novel numerical scheme for ECG Signal denoising with low computational cost, in terms of memory and time consuming. It removes all artifacts on the ECGs that have the spectrum frequency range known and limited. Our approach is based on the analysis of the signal in the Fourier domain, but it does not require a direct application of the (Fast) Fourier Transform. With respect to other methods, we compute the solution with only a few floating point operations. Moreover, it supports suitable fitting conditions that allow to the scheme the local denoising and hence the visualization of the denoised signal in real time. The other filters, reported above, can work very well on the entire signal, but for a local denoising, they need of appropriate fitting conditions on two consecutive parts of the signal that are contained in two successive data packets sent by the wearable sensor. This feature makes the scheme suitable for direct implementation in applications on mobile devices, dedicated to the real time filtering of biomedical signals. In order to test the algorithm, we report the performance metrics achieved by applying the proposed methodology to some records of the database PhysioNet [12], that offers a large collection of recorded physiological ECG signals. The paper is organized as follows: in section 2 we give some preliminary mathematical considerations; section 3 is devoted to the numerical scheme; in section 4 we report the numerical experiments and, finally, in section 5

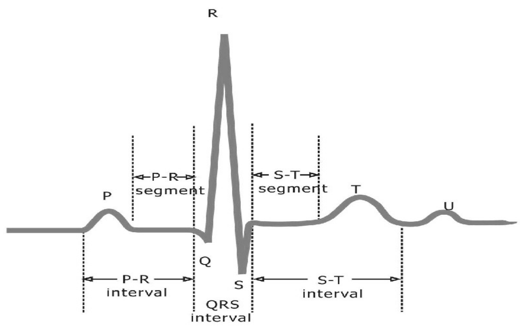

Figure 1: An example of a standard ECG waveform where are shown PR, QRS, ST intervals, P,Q,R,S,T and U waves.

we draw our conclusions.

2 Mathematical Preliminaries

In this section we give the mathematical preliminaries of a scheme for the filtering of digital signals with a computational cost of $O(n)$ floating point operations. This scheme is based on an approximation of a continuous convolution with a suitable convolution function h. Let s_0 denote a real function such that

$$s_0 = s + \epsilon$$

with s the original signal and ϵ a noise function. In computer vision, researchers as in (e.g. [4]) use the convolution of s_0 with a function h, Lebesgue integrable, to obtain a denoised function s_h, i.e.:

$$s_h(t) = [h * s_0](t) = \int_{-\infty}^{+\infty} h(t-x)s_0(x)dx, \quad \forall t \in \mathbb{R}. \tag{1}$$

The focus of this section is to determine suitable properties for the function h in order to determine a denoising scheme. The scheme has to eliminate noises ϵ with frequency spectrum that wanders in a limited range. To this aim, we use the following mathematical tools:

- the Fourier Transform $\mathcal{F}$ of a signal f:

$$F(\omega) = \mathcal{F}(f)(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x)e^{-i\omega x}dx, \quad \forall \omega \in \mathbb{R}; \tag{2}$$

- the Fourier anti-Transform $\mathcal{F}^{-1}$ of F:

$$f(t) = \mathcal{F}^{-1}(F)(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} F(x)e^{itx}dx, \quad \forall t \ \in \mathbb{R}; \tag{3}$$

- the convolution theorem

$$\mathcal{F}(h * f) = \mathcal{F}(h) \cdot \mathcal{F}(F) = H \cdot F. \tag{4}$$

The main idea of this work is to find a suitable convolution kernel h, to denoise s_0, starting from its Fourier Transform $H = \mathcal{F}(h)$ in the Fourier domain. Now, let us suppose that

$$S_h = \mathcal{F}(s_h), \quad S_0 = \mathcal{F}(s_0), \quad S = \mathcal{F}(s) \quad \text{and} \quad \mathcal{E} = \mathcal{F}(\epsilon).$$

If we assume that the Fourier Transform $H = \mathcal{F}(h)$ of h is such that:

$$\begin{cases} H \cdot S = S, \\ H \cdot \mathcal{E} = 0, \end{cases} \tag{5}$$

then, it can be easily proved that it holds that:

$$s_h = h * s_0 \equiv s. \tag{6}$$

Observing that, if the original and noise signals, s and ϵ, have the property:

$$supp\, S \cap supp\, \mathcal{E} = \varnothing, \tag{7}$$

in the Fourier domain, then, it is possible to obtain an infinite class of functions that satisfy condition (5). Then, in these hypotheses, the original signal s can be completely restored by the convolution in (6). Nevertheless, as shown in the next example, a satisfying solution s_h (an approximation of s) can be achieved without assuming (7), and by filtering the signal s_0 by means of a certain convolution kernel.

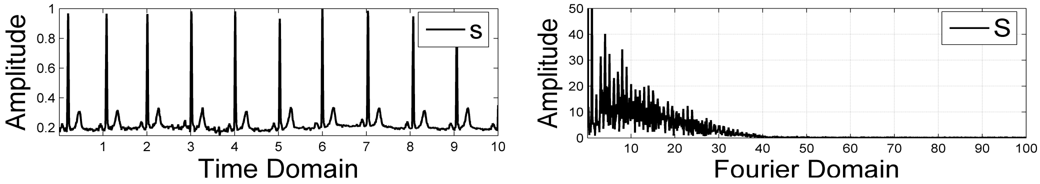

Figure 2: Left: an ECG signal s. Right: $\mathcal{F}-$Transform $S = \mathcal{F}(s)$ of s

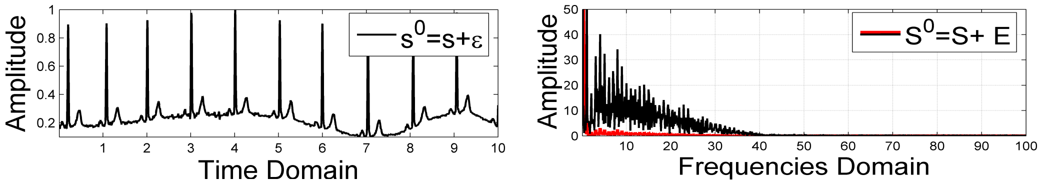

Figure 3: Left: the noisy ECG signal $s_0 = s + \epsilon$. Right: $\mathcal{F}-$Transform $S_0 = \mathcal{F}(s_0)$ of s_0 , sum of $S = \mathcal{F}(s)$ (black line) and of $\mathcal{E} = \mathcal{F}(\epsilon)$ (red line)

Now, let s be an original ECG signal (Figure 2, left) and $s_0 = s + \epsilon$ (Figure 3, left) the noisy signal, obtained by the noise function ϵ. Here we take ϵ as the Base Line Wander noise (e.g.[2]) Assuming that the interval $[\mu - \sigma,\ \mu + \sigma]$ contains the support of the Fourier Transform $\mathcal{E}$, i.e. $supp\,\mathcal{E} \subseteq [\mu - \sigma,\ \mu + \sigma]$, then we consider a convolution kernel $h = \mathcal{F}^{-1}(H)$, where H is set as:

$$H(\omega) = \begin{cases} 0, & \text{if } \omega \in [\mu - \sigma,\ \mu + \sigma]; \\ 1, & otherwise. \end{cases} \tag{8}$$

With these assumptions, a way of obtaining a denoised signal s_h is given by the following steps:

(i) $\mathcal{F}$-Transform s_0, to obtain $S_0 = \mathcal{F}(s_0) = S + \mathcal{E}$ (S and S_0 are shown, respectively, on the right-hand side of Figure 2 and of Figure3);

(ii) define the function H as in (8);

(iii) multiply H for S_0, to achieve $S_h = H \cdot S_0$ (left of Figure 4);

(iv) $\mathcal{F}^{-1}$-Transform $S_h = H \cdot S_0$, to extract from s_0 the signal $s_h = \mathcal{F}^{-1}(S_h)$ (Figure 4, right).

All these figures have been generated by a `Matlab` code (Code 1) that uses the Fast Fourier Transform (FFT) and Inverse Fast Fourier Transform (IFFT) (e.g. [26]), in order to implement steps (i)–(iv). This code takes into account the symmetry of $supp\,\mathcal{E}$ in the Fourier domain, when ϵ is a periodic function. In particular in Code 1, the parameters μ, σ of (8) are set as $\mu = 0.4\,Hz$ and $\sigma = 0.3\,Hz$.

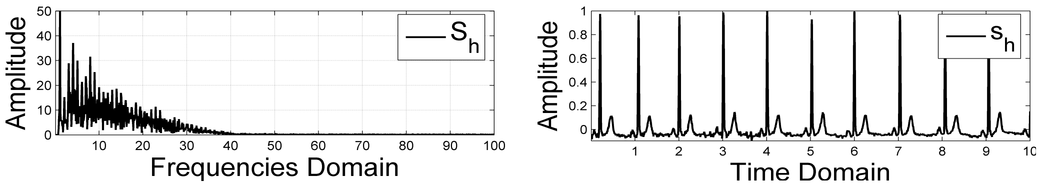

Figure 4: Left: $\mathcal{F}-$Transform $S_h = \mathcal{F}(s_h)$. Right: the restored signal s_h

```
function s_h = FFT_Filter(s_0,mu,sigma);% Inputs: % s_0 vector noised input data
fl=(mu-sigma)/fs ; fu=(mu+sigma)/fs %normalized lower and upper frequency to cut
S_0 = fft(s_0,n,2); H=ones(1,n);
k =floor(fl*n):floor(fu*n);  H(1,k) = 0;  H(1,n-k+2) = 0;
S_h(1,:)=H(1,:).*S_0(1,:);  s_h= real(ifft(S_h,n,2));
```

Code 1: Matlab Code of a FFT Filter for the Removal of the Base Line Wander in ECG signals

We highlight that the right of Figure 3 indicates that $supp\, S \cap supp\, \mathcal{E} \neq \varnothing$, but after applying the Code 1, in which H is set as 0 also on $supp\, S \cap supp\, \mathcal{E}$, the signal s_h (right of Figure 4) can anyway be considered an accurate approximation of s (Figure 2, left). However, the mathematical form of H in (8), and its discontinuities, prevent us from determining our numerical scheme. Therefore, for our purpose, as we will show in the next section, instead of H, we will use a function $\tilde{H}$ that emulates the properties of H. This function is defined as:

$$\tilde{H}(\omega) = \frac{(\omega-\mu)^2}{2\sigma^2 + (\omega-\mu)^2}, \quad \forall \omega \in \mathbb{R}. \tag{9}$$

and is a rational approximation of the function:

$$G(\omega) = 1 - e^{(-(\omega-\mu)^2/2\sigma^2)}, \quad \forall \omega \in \mathbb{R}. \tag{10}$$

Notice that $\tilde{H}$ is obtained from G, by taking the first two terms in the Taylor expansion of the following exponential function:

$$e^{((\omega-\mu)^2/2\sigma^2)} = \sum_{i=0}^{+\infty} \frac{1}{i!} \left(\frac{(\omega-\mu)^2}{2\sigma^2} \right)^i \tag{11}$$

The functions $\tilde{H}$ and G have the following properties:

1. $0 \leq G(\omega), \tilde{H}(\omega) < 1, \quad \forall\, \omega \in \mathbb{R}$;
2. $\tilde{H}(\mu) = G(\mu) = 0$;
3. $\lim_{\omega \to \pm\infty} \tilde{H}(\omega) = \lim_{\omega \to \pm\infty} G(\omega) = 1$;
4. $\tilde{H}(\omega)$ and $G(\omega)$ are symmetrical with respect to the axis $\omega = \mu$;
5. $\tilde{H}(\mu \pm \sigma) = 1/3$;
6. $\tilde{H}(\omega) = \tilde{H}_l(\omega) \cdot \tilde{H}_r(\omega), \quad \forall \omega \in \mathbb{R}$, where we set:

$$\tilde{H}_l(\omega) = \frac{-i\omega + i\mu}{-i\omega + (\sqrt{2}\sigma + i\mu)}, \qquad \tilde{H}_r(\omega) = \frac{i\omega - i\mu}{i\omega + (\sqrt{2}\sigma - i\mu)}. \tag{12}$$

Observe that it is still possible to compute the denoised signal $s_{\tilde{h}}$ by means of Code 1, replacing H in (8) by $\tilde{H}$ in (9). The choice of function $\tilde{H}$ in (9) can determine the noisy ϵ, even if its frequency spectrum is wandering in the interval $[\mu-\sigma, \mu+\sigma]$. Moreover, with this function, it is possible to provide a numerical scheme to obtain the denoised function $s_{\tilde{h}}$ from s_0 as shown in next section.

3 A novel $\mathcal{O}(n)$ Numerical Scheme

In this section, we introduce the derivation of our scheme for the denoising of digital signals. This algorithm is based on the Infinite Impulse Response (IIR) Gaussian Recursive Filter of [27]. It reduces the effects of additive noise functions ϵ on the original signal s, when $supp\, \mathcal{E} \subset [\mu-\sigma, \mu+\sigma]$. In terms of floating point operations, this algorithm is faster than FFT: as we will show later, it has a computational cost of $\mathcal{O}(n)$, while FFT has a cost of $n\log(n)$.
As a preliminary remark, we observe that if $S_0 = \mathcal{F}(s_0)$ and if $\tilde{h} = \mathcal{F}^{-1}(\tilde{H})$, with $\tilde{H}$ defined as in (9), then for the function $s_{\tilde{h}} = \tilde{h} * s_0$ it holds that

$$s_{\tilde{h}} = \tilde{h}_l * (\tilde{h}_r * s_0) \tag{13}$$

where functions $\tilde{h}_l$ and $\tilde{h}_r$ are defined as

$$\tilde{h}_l = \mathcal{F}^{-1}(\tilde{H}_l) \quad \text{and} \quad \tilde{h}_r = \mathcal{F}^{-1}(\tilde{H}_r),$$

and functions $\tilde{H}_l$ and $\tilde{H}_r$ are as in (12). In order to determine a numerical scheme, we need to sample the signals s, s_0. From now on, we will consider the discrete signals:

$$\vec{s}_0 = (s_0[1], \ldots, s_0[n]), \quad \vec{s} = (s[1], \ldots, s[n]) \quad \text{and} \quad \vec{\epsilon}_0 = (\epsilon[1], \ldots, \epsilon[n])$$

obtained from s_0, s, ϵ, by using an uniform discretization with stepsize τ, i.e.:

$$s_0[j] = s_0(j\tau), \quad s[j] = s(j\tau), \quad \epsilon[j] = \epsilon(j\tau), \qquad j = 1, ..., n. \tag{14}$$

It is well known that for the frequency range of discrete signals it holds that:

$$-\frac{\pi}{\tau} \le \omega \le \frac{\pi}{\tau} \quad \Longleftrightarrow \quad -\frac{\phi_d}{2} \le \phi \le \frac{\phi_d}{2} \tag{15}$$

where $\phi_d = 1/\tau$ and $\omega = 2\pi\phi$. Our scheme is based on the discretization of the continuous scheme:

$$\mathcal{S}_{\tilde{h}} = \tilde{H}_l \cdot \tilde{H}_r \cdot S_0, \tag{16}$$

where $\tilde{H}_l$ and $\tilde{H}_r$ are defined as in (12). The poles of these functions lie in the left-half plane of complex plane hence represent respectively cause and anti-cause stable differential equations for continuous signals that can be transformed into causal and anti-causal difference equations for discrete signals by means of standard technique; classic methods are bilinear transform, finite differences, the zero-pole matching method and others (e.g. [17]). For our scheme we have used the zero-pole matching method. This approach has the advantage of transforming stable differential equations into stable difference equations and of not using approximations like the others. Let $\vec{s}_0$ be a discrete signal with sampling step τ, by applying the zero-pole matching method (e.g. [17]) to continuous scheme $S_{\tilde{h}} = \tilde{H}_l \cdot \tilde{H}_r \cdot S_0$, we obtain the following forward and backward denoising numerical scheme.

Denoising Numerical Scheme

$$\boxed{\begin{aligned} p_{\tilde{h}}[j] &= b_0 s_0[j] + b_1 s_0[j-1] + b_2 s_0[j-2] + \\ &\qquad a_1 p_{\tilde{h}}[j-1] + a_2 p_{\tilde{h}}[j-2] \quad j = 1, ..., n \ : +1 \\ s_{\tilde{h}}[j] &= b_0 p_{\tilde{h}}[j] + b_1 p_{\tilde{h}}[j+1] + b_2 p_{\tilde{h}}[j+2] + \\ &\qquad a_1 s_{\tilde{h}}[j+1] + a_2 s_{\tilde{h}}[j+2] \quad j = n, ..., 1 \ : -1 \end{aligned}} \tag{17}$$

where the recursive scheme coefficients in (17) are:

$$b_0 = 1, \quad b_1 = -2\cos(\mu\tau), \quad b_2 = 1 \quad and \quad a_1 = 2e^{-\sqrt{2}\sigma\tau}\cos(\mu\tau), \quad a_2 = -e^{-2.\sqrt{2}\sigma\tau}. \tag{18}$$

The computational cost of the forward and backward difference scheme in (17) is:

$$\mathcal{T}(n) = 18\, n\, t_{calc}. \tag{19}$$

where n is the size of the discrete signals $\vec{s}_0, \vec{p}_{\tilde{h}}$ and $\vec{s}_{\tilde{h}}$ and t_{calc} is the time for a floating point operation. To close equations in (17), at the borders, we have to fix the initial conditions: we assume that signals $\vec{s}_0$, $\vec{p}_{\tilde{h}}$ and $\vec{s}_{\tilde{h}}$ exist and we set a constant value for indexes $j < 1$ and $j > n$. Then the border conditions for $\vec{s}_0$ are the following: $s_0[j] = s_0[1]$ for all $j < 1$, then for $\vec{p}_{\tilde{h}}$, it holds that $p_{\tilde{h}}[j] = (b_0 + b_1 + b_2)s_0[1]/(1 - a_1 - a_2)$ $\forall j < 1$, i.e. the steady-state response to an infinite stream of $s_0[1]$ value using the first equation in (17); similarly, $\forall j > n$, the $p_{\tilde{h}}[j]$ assumes a constant value $p_{\tilde{h}}[n]$, then $s_{\tilde{h}}[j] = (b_0 + b_1 + b_2)p_{\tilde{h}}[n]/(1 - a_1 - a_2)$ for all $j > n$, i.e. the steady-state response to an infinite stream of $p[n]$ value using the second equation in (17). Hence, to complete the statements in (17) we fix the following heuristic initial conditions for the forward and backward procedures:

$$\boxed{\begin{array}{ll} \textit{forward conditions} & \textit{backward conditions} \\ p_{\tilde{h}}[-1] = p_{\tilde{h}}[0] = \dfrac{(b_0 + b_1 + b_2)}{(1 - a_1 - a_2)}s_0[1] & s_{\tilde{h}}[n+2] = s_{\tilde{h}}[n+1] = \dfrac{(b_0 + b_1 + b_2)}{(1 - a_1 - a_2)}p_{\tilde{h}}[n] \end{array}} \tag{20}$$

For the local denoising, that allows the visualization of the denoised signal in real time, we have used a methology based on the work of Triggs et al. [25] for the determining of the fitting conditions between two consecutive parts of the signal. Here we have not reported these discussions because they require complex mathematical notions but we will discuss them more in detail in future work. We conclude the section by giving a possible scheme of the algorithm that uses the forward and backward equations in (17). The algorithm can be used for the denoising of a discrete signal $\vec{s}_0 = \vec{s} + \vec{\epsilon}$, where the frequency spectrum range of $\vec{\epsilon}$ is known. The scheme of the proposed denoising algorithm, from now on indicated as the Recursive Filter (RF), is the following:

Algorithm 1 Scheme of the Recursive Filter (RF)

Input: $\vec{s}_0$ μ, σ Output: $\vec{s}_{\tilde{h}}$

Compute b_0, b_1, b_2, a_1 and a_2 as in (18)
  $p_{\tilde{h}}[-1] := p_{\tilde{h}}[0] := ((b_0 + b_1 + b_2)/(1 - a_1 - a_2))s_0[1]$
  for j=1,2,...,n
    $p_{\tilde{h}}[j] := b_0 s_0[j] + b_1 s_0[j-1] + b_2 s_0[j-2] + a_1 p_{\tilde{h}}[j-1] + a_2 p_{\tilde{h}}[j-2]$
  endfor
  $s_{\tilde{h}}[n+2] := s_{\tilde{h}}[n+1] := ((b_0 + b_1 + b_2)/(1 - a_1 - a_2))p_{\tilde{h}}[n]$
  for j=n,n-1,...,1
    $s_{\tilde{h}}[j] = b_0 p_{\tilde{h}}[j] + b_1 p_{\tilde{h}}[j+1] + b_2 p_{\tilde{h}}[j+2] + a_1 s_{\tilde{h}}[j+1] + a_2 s_{\tilde{h}}[j+2]$
  endfor
Return $\vec{s}_{\tilde{h}}$

4 Numerical Experiments on the ECGs

In this section, we compared, in terms of accuracy and computational cost measures, the results of RF to them of a method that exploits the FFT as in Code 1, of a first order zero-phase lowpass filter (LPF) (e.g. [17]) and of a single stage of median or moving average filtering (BPF) (e.g. [7, 11]). In our experiments, we have used data from the Physionet Long-Term ST

Database [12]. The Long-Term ST Database contains 86 lengthy ECG recordings of 80 human subjects, chosen to exhibit a variety of events of ST segment changes, including ischemic ST episodes, axis-related non-ischemic ST episodes, episodes of slow ST level drift, and episodes containing mixtures of these phenomena. The database was created to support development and evaluation of algorithms capable of accurately differentiating of ischemic and non-ischemic ST events, as well as basic research into mechanisms and dynamics of myocardial ischemia. Then a pre-processing phase is due to eliminate the additional artifact on ECGs, before using these algorithms, for efficient distinction between physiological and pathological events.

The ECG signals used, from the Long-Term ST database, last 3600 seconds (s) and are: s20011, s20051, s20061, s20071 and s20081 and s20121. These recordings were sampled at 250 Hz using 11-bit A/D converters. We have processed in this paper only the Physionet recordings with synthetic Base Line Wander noise added. Base Line Wander is caused by respiration or patient movement which create problems in the detection of peaks (see Figure 1 and center of Figure 5). Due to wander the T peak would be higher than the R peak and might be detected as an R peak instead. The amplitude variation is 15% of the peak to peak ECG amplitude. It is normally considered below 1 Hz. The FFT method, LPF and BPF algorithms above represent in literature the fastest and most accurate approaches for the the denoising of an ECG with a Base Line Wander noise.

Let $\vec{s}_0 = \vec{s} + \vec{\epsilon}$ with $\vec{s}$, $\vec{s}_0$ and $\vec{\epsilon}$ described in Section 3. The signal s_0 is shown in the left of Figure 5. In the right of the same picture, we have reported $\vec{s}_{\tilde{h}}$, the RF application to $\vec{s}_0$ choosing $\mu = 0.25$ Hz and $\sigma = 0.9$. For other kinds of artifacts μ and σ have to be chosen according to spectrum frequency range of the noises. The first impression is that the RF reconstructs quite successfully the signal $\vec{s}$ also in a part of the ECG where there is a pathological (ischemic) event (red zone of Figure 5).

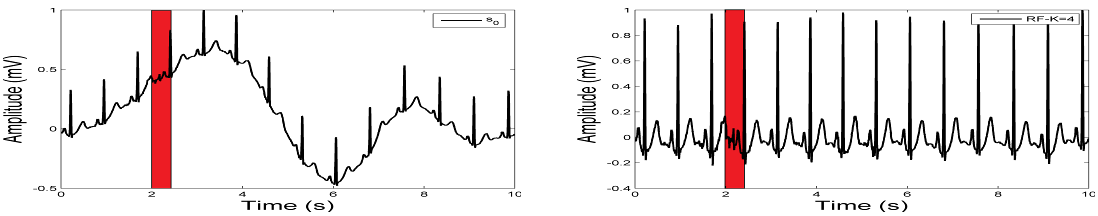

Figure 5: Left: The figure represents 10 seconds of a noisy signal $\vec{s}_0$. Right: The figure represents the denoised signal $\vec{s}_{\tilde{h}}$

As in [14], we quantify the denoising performance in terms of the Signal-to-Noise ratio (SNR) (in decibels):

$$SNR = 10\log_{10}\left(\frac{\sum_{i=1}^{n}\left(s_0[i]-s[i]\right)^2}{\sum_{i=1}^{n}\left(s_f[i]-s[i]\right)^2}\right) \tag{21}$$

In (21), $\vec{s}_f$ is one of the denoised signals obtained by means of the filter above and n is the length of $\vec{s}$, $\vec{s}_0$ and $\vec{s}_f$. We highlight that here we have assumed that the Physionet signals $\vec{s}$ are the true signals; actually, these signals also contain noise, which the metric above neglects. In Table 1, we have reported the SNR measures, varying the ECGs chosen and varying the filters considered. First of all, we can observe from Table 1, that the most accurate filter is, in any case, the FFT method, with in second position the RF. Of the left-hand side of Figure 6, we show the average results in terms of execution time and memory usage of the examined filters, for the denoising of ECG s20011 with size n=900000. The experiments have been carried out using an Asus CPU Intel(R) Core(TM) i7-4510U CPU 2.00 GHZ -2.60 GHZ, RAM 6 GB. Figure

Table 1: Signal-to-Noise ratio (SNR) (in decibels)

SNR	$s20011$	$s20051$	$s20061$	$s20071$	$s20081$	$s20121$
FFT	15.73	14.15	17.72	14.44	13, 85	17.65
LPF	13.96	12.75	15.71	13.64	13, 21	14.27
BLF	10.37	11.18	10.90	9.54	10, 79	11.35
RF	14.39	13.33	15.88	13.69	13, 17	14.41

6 (left) shows that RF and LPF have the lowest time while the RF and the FFT method exploit the lowest amount of memory. However, for a filter based on the Fourier Transform, we need to use efficiently FFT-libraries [1], while our code requires the implementation of only few code lines to remove the main artifacts from ECG recordings choosing suitable μ and σ parameters. We think this feature the most important of this algorithm. Hence taking into consideration, the SNR measure in Table 1 and the computational cost tests in the left of Figure 6 then RF has the possibility of direct implementation on mobile computing or wearable sensors (equipped with CPU). for the denoising of ECG signals. Finally, we report in the right of Figure 6 a

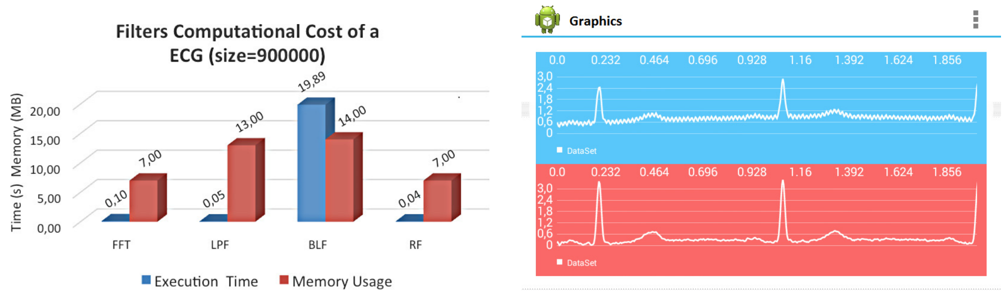

Figure 6: Left: The figure shows the computational cost (execution time and memory usage) of FFT, LPF, BPF and RF to denoise an ECG (s20011) with a size of 900000 samples. Right: Screenshot of an Android application for the de-noising of an ECG that implements the RF scheme.

screenshot of an Android application for the de-noising of an ECG based on RF. It proves to be very fast on many devices of the latest generation, also for long ECGs recordings.

5 Conclusions

In this paper we have described the development and implementation of a scheme (RF) for ECG Signal Denoising applicable in the contest of the real-time health monitoring, where the signal processing efficiency is strictly demanded.

Numerical experiments on some ECGs from the Physionet Long-Term ST Database have demonstrated that our RF can significantly reduce the total computational cost of denoising compared to other efficient filters, while maintaining the same level of accuracy and using only few code lines. In addition, we provide the theoretical development of the new scheme, based on the study in [27], who formulated the RF in the context of signal processing to eliminate high frequency noise. We have adapted this for ECG signal denoising, providing a description of the process to obtain the RF coefficients for the main artifacts of ECG recordings. The RF is fast and exhibits a computational cost of only $\mathcal{O}(n)$. Therefore, we can implement it on mobile computing devices to achieve improvements in e-health care field.

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